Patrick Pei

CS3252-01

Due: 2/16/17

Assignment 6

*4.1.1 b

Prove that $L_{balanced}$ which consists of all strings with balanced parentheses is not a regular language.

It is clear that $w = (^n)^n$ is in $L_{balanced}$. w can be expressed as xyz and by the pumping lemma,

1. $y \neq \varepsilon$
2. $|xy| \leq n$
3. for $k \geq 0$, $xy^kz \in L$.

Since $|xy| \leq n$ and xy is at the front, xy clearly consists of all left parentheses. The pumping lemma says that $xz \in L$ if L is regular but since in the case of $k=0$, xz has fewer left parentheses since $y \neq \varepsilon$ which means there can be no more than n-1 left parentheses among xz. By contradiction, $L_{balanced}$ is not regular.

*4.1.1 d

Prove $L = \{0^n 1^m 2^n \mid n \text{ and } m \text{ are integers}\}$ is not regular

It is clear that for $m = 0$, $w = 0^n 2^n$

By the pumping lemma, $w = xyz$, if $k=0$ it can be seen in the same way as above. $|xy| \leq n$ and $y \neq \varepsilon$, xz has less 0's than 2's and contradicts $xz \in L$.

*4.1.1 e

Prove $L = \{0^n 1^m \mid n \leq m\}$ is not regular

for $n = m$, it is clear that $w = 0^n 1^n \in L$.

By the pumping lemma, $w = xyz$, if $k=0$ it can be seen in the same way as above. $|xy| \leq n$ and $y \neq \varepsilon$ so xz has less 0's than 1's and contradicts $xz \in L$. Thus, L is not a regular language.

*4.1.2b

Prove that $L = \{0^n \mid n \text{ is a perfect cube}\}$ is not regular

It is clear that $w = 0^{n^3} \in L$.

w can be expressed as $w = xyz$ where $1 \le |y| \le n$

Thus for xy^kz where $k = 2$, $2 \le |y^2| \le 2n$ and the new length of w increases by at most n. However, the next perfect cube, $(n+1)^3 = (n+1)(n^2+2n+1)$

$$= n^3 + 2n^2 + n + n^2 + 2n + 1$$

$$= n^3 + 3n^2 + 3n + 1$$

Thus, the length does not increase by $3n^2 + 3n + 1$ and thus $xy^2z \notin L$ proving L is not a regular language.

*4.1.2c

Prove that $L = \{0^n \mid n \text{ is a power of } 2\}$ is not regular

It is clear that $w = 0^{2^n} \in L$.

w can be expressed as $w = xyz$ where $1 \le |y| \le n$

For xy^kz where $k = 2$, the next accepted string is $0^{2^{n+1}}$ which means the increase in length by pumping y must be equal to 2^n which is clearly impossible since pumping y once can only increase the length by n. Thus, since 2^n is exponential and n is linear, these growth rates are not equal and therefore L is not regular.

*4.1.2e

Prove that L: set of strings of 0's and 1's that are of the form ww

It is clear that $w = 0^n 1^n 0^n 1^n \in L$. Since $|xy| \le n$, and $y \ne \varepsilon$, y consists of 1 or more 0's but not more than n. In other words: $1 \le |y| \le n$. Thus, it can be seen that xz consists of less than n 0's and then $1^n 0^n 1^n$ which clearly is not of the form ww. Therefore, a contradiction is formed and L is proven to not be a regular language.

* 4.1.2 f

Prove that the language L = the set of strings of 0's and 1's that are of the form ww^R is not regular.

It is clear that $w = 0^n 1^n 1^n 0^n$ is in L. Since $|xy| \leq n$, and $y \neq \varepsilon$, it is clear that x consists of only 0's and $|x| < n$, then xz is clearly not part of L since it starts with fewer than n 0's and is followed by $1^n 1^n 0^n$. Thus, a contradiction is formed and L is not regular.

* 4.1.2 g

Prove that the language L = the set of strings of 0's and 1's of the form $w\bar{w}$ is not regular.

It is clear that $w = 0^n 1^n$ is in L. By the pumping lemma, w can be expressed as $w = xyz$ and since $|xy| \leq n$, with $y \neq \varepsilon$, x has fewer than n 0's and consists of only n 0's. Thus, xz is not in L since there are fewer 0's than 1's. Therefore, a contradiction is formed and L is proven to not be regular.

* 4.2.3

Since we know regular languages are closed under reversal and the quotient operation of 4.2.2, we can manipulate L to show that if L is regular, so is $a \backslash L$. For the quotient operation, L/a is the set of strings w such that wa is in L, meaning that the selected strings ends in a. To show that $a \backslash L$ is regular, it is clear to remove from the beginning is the same as removing from the end of the reversed. In other words, $a \backslash L = (L^R/a)^R$. Thus, since regular languages are closed under reversal and the quotient operation, $a \backslash L$ is regular if L is.

*4.2.5

a. $\frac{d(R+S)}{da} = a \backslash L(R+S)$

$= a \backslash (L(R) \cup L(S))$

$= a \backslash L(R) \cup a \backslash L(S)$

$= \frac{dR}{da} + \frac{dS}{da}$

b. If $\varepsilon \notin L(R)$, then $\frac{dRS}{d\,\text{letter}} = \frac{dR}{d\,\text{letter}} S$

else:

$$\frac{dRS}{d\,\text{letter}} = \frac{dR}{d\,\text{letter}} S + \frac{dS}{d\,\text{letter}}$$

c. $\frac{d(R^*)}{da} = \frac{d(RR^*)}{da} = \frac{dR}{da} R^* + \varepsilon$

d. $\frac{d((0+1)^* 011)}{d0}$ $R = (0+1)^*$

$S = 011$

$$\frac{d(RS)}{d0} = \frac{dR}{d0} S + \frac{dS}{d0}$$

$$= \frac{d((0+1)^*)}{d0} 011 + \frac{d(011)}{d0} +$$

$$= \left(\frac{d(0+1)}{d0} (0+1)^* + \varepsilon \right) 011 + 11$$

$$= \left(\left(\frac{d0}{d0} + \frac{d1}{d0} \right) (0+1)^* + \varepsilon \right) 011 + 11$$

$$\left((0+1)^* + \varepsilon \right) 011 + 11$$

$$(0+1)^* 011 + 011 + 11$$

$$\frac{d\left((0+1)^* 011\right)}{d1} = \qquad R = (0+1)^* \quad S = 011$$

$$\frac{d(RS)}{d1} = \frac{dR}{d1} S + \frac{dS}{d1}$$

$$= \frac{d\left((0+1)^*\right)}{d1} 011 + \frac{d(011)}{d1}$$

$$= \left(\frac{d(0+1)}{d1}(0+1)^* + \varepsilon\right) 011 + \varepsilon$$

$$= \left(\left(\frac{d0}{d1} + \frac{d(1)}{d1}\right)(0+1)^* + \varepsilon\right) 011 + \varepsilon$$

$$= \left((\varepsilon + \varepsilon)(0+1)^* + \varepsilon\right) 011 + \varepsilon$$

$$= (0+1)^* 011 + 011 + \varepsilon$$

$$= (0+1)^* 011 + 011$$

e. L with no strings that start with 0

f. the languages L for which $\frac{dL}{d0} = L$ must be of the form

$L(0^*)M$ where M does not start in 0's