CS-3252

Section 01

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Assignment #2

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Exercises

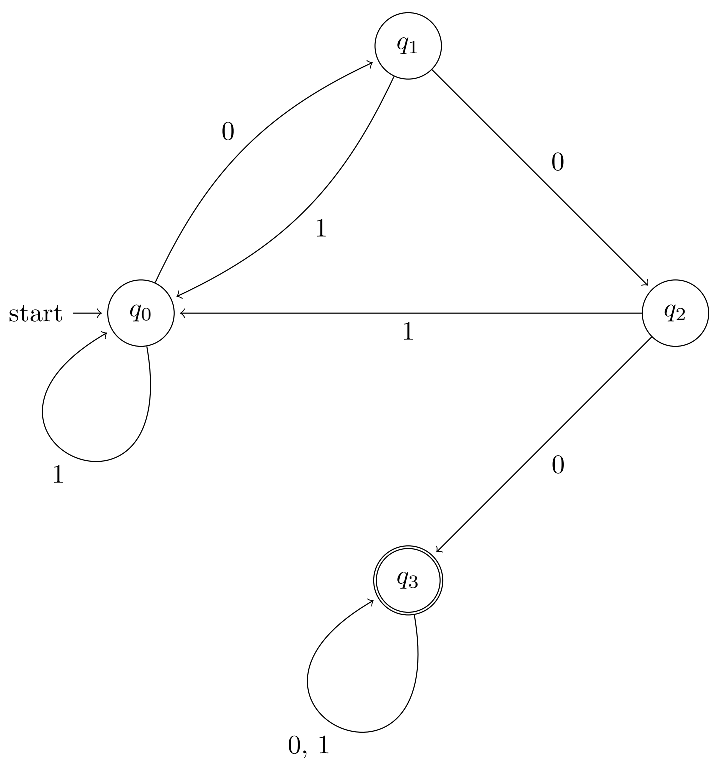
- Exercise 2.2.4

(b) The set of all strings with three consecutive 0's (not necessarily at the end).

Assuming A is the name of this DFA, Q is its set of states, Σ is its input symbols, δ is its transition function, q_0 its start state, and F is its set of accepting states.

$$A = (Q, \Sigma, \delta, q_0, F)$$

$$A = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, q_3)$$

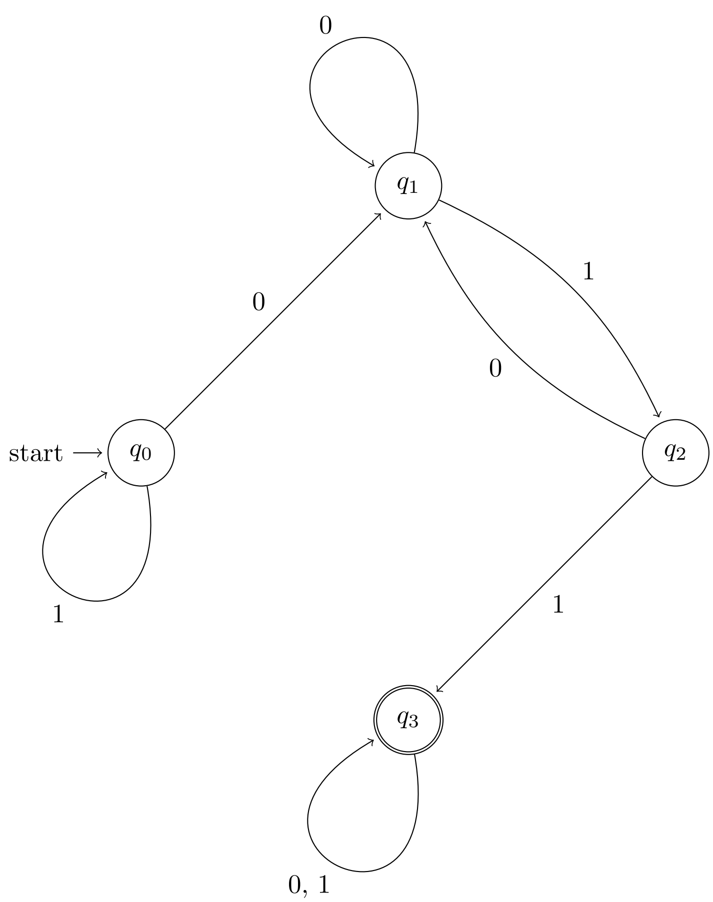


(c) The set of strings with 011 as a substring.

Assuming A is the name of this DFA, Q is its set of states, Σ is its input symbols, δ is its transition function, q_0 its start state, and F is its set of accepting states.

$$A = (Q, \Sigma, \delta, q_0, F)$$

$$A = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, q_3)$$



- Exercise 2.2.7

  Let A be a DFA and q a particular state of A, such that δ(q, a) = q for all input symbols a. Show by induction on the length of the input that for all input strings w , δ(q, w) = q .

  Suppose that for any input string w, w is a string of the form xa, where a is the last symbol of w, and x is the string consisting of all but the last symbol. Thus, since the basis of $\hat{\delta}(q, \epsilon) = q$ is valid because reading in no inputs while in state q obviously remains in state q, any string can be built up of a finite amount of characters. Each of these strings can be subsequently reduced and ultimately will arrive at the base case. In other words, for all strings of length w + 1, the inductive hypothesis holds true.

  Basis: $\hat{\delta}(q, \epsilon) = q$, where the length of w is zero

  Induction: If w is a string with length greater than zero, and is broken down as before into xa where a is the last symbol and x is the string consisting of all but the last symbol, then $\hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a)$ by definition. Finally, since it is given that $\delta(q, a) = q$, this is equivalent to $\hat{\delta}(q, w) = \hat{\delta}(q, x)$ and subsequent reducing of the string length by induction will result in $\hat{\delta}(q, w) = q$

- Exercise 2.2.11

  The language accepted by this DFA is any input of ones and zeroes that does not contain two consecutive zeroes.

  Basis: $\hat{\delta}(q, \epsilon) = q$, where the length of w is zero. Clearly a string of length zero cannot contain two consecutive zeroes. This also means that $\hat{\delta}(A, w) = C$ iff w contains two consecutive zeroes as well as $\hat{\delta}(A, w) = A$ iff w is empty (as defined before) or is of the form x1 iff x does not contain two consecutive zeroes because 1 will result in A, and $\hat{\delta}(A, w) = B$ iff w is of the form x0 iff x does not end in a zero because $\delta(A, 0) = B$.

  Induction: If w is a string with length greater than zero, and is broken down as before into xa where a is the last symbol and x is the string consisting of all but the last symbol, then $\hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a)$ by definition. Since from inspection of the DFA's transition table and a hand-drawn diagram, it is clear that the input up until the current at state A will not have ended in a zero (it would have been a one from B or empty) and similarly that the input up until the current at state B would have ended in a zero. From here it is enough to inductively prove all cases from an accepted state and show that the only case that is not accepted is one with two consecutive zeroes. Since two numbers from the set of {0, 1} can only form four total possibilities, namely 00, 01, 10, 11, each of these cases can be examined. If the automaton is at state A, 00 obviously sends it to an unaccepted state, 01 is equivalent to $\hat{\delta}(A, \text{“01”}) = \delta(B, 1) = A$ as mentioned in the base cases. For the case of 10, $\hat{\delta}(A, \text{“10”}) = \delta(A, 0) = B$. Lastly, the case of 11 clearly is $\hat{\delta}(A, \text{“11”}) = \delta(A, 1) = A$. Thus, from A, the only input to result in an unaccepted state is that of 00. From B, there are only two cases to be examined, name 1 and 0 since it is known that the input up until then to ever arrive at B must have ended in a zero. Thus, an input of 1 sends the automaton back to state A as previously described and an input of 0 is sent to the unaccepted state, state C. Evidently, the only input combination to send the automaton to the unaccepted state again was 00. By induction, all strings of length w + 1 can be applied in the same way and it is evident that all inputs from state C are to be disregarded since by inspection state C can only transition to itself.