

1) Show that 3 of the following 4 problems are in NP.

- a) Given a graph  $G$  and a number  $k$ , does  $G$  have a spanning tree of cost less than or equal to  $k$ ?
- b) Given a graph  $G$  and a number  $k$ , does  $G$  have no spanning tree of cost less than or equal to  $k$ ?
- c) Given a CNF expression  $E$  and a number  $k$ , is there an assignment which makes at least  $k$  clauses true?
- d) Given a CNF expression  $E$  and a number  $k$ , is there no assignment which makes at least  $k$  clauses true?

2) The simple  $k$ -path problem takes a graph  $G$  and a number  $k$ , and asks whether  $G$  has a path of at least  $k$  edges such that no vertex appears on the path more than once. Show that the simple  $k$ -path problem is NP-complete.

3) Assume that it is NP-complete to determine whether a graph can be colored with 3 colors. Show that it is NP-complete to determine whether a graph can be colored with 4 colors.

4) Show that it is NP-complete to determine whether a graph has a clique consisting of exactly half of the vertices in the graph.