Homework 1 CS 350 Due Jan 27

Do not work together with anyone else, except to clarify the problem. Any work which is not entirely your own MUST be cited. You may assume all functions are positive. Note that if not otherwise specified, running time of an algorithm is always defined as worst case running time.

1) Prove or disprove: For every fixed positive integer k, $1^k+2^k+\ldots+n^k$ is $O(n^{k+1})$.

2) Prove or disprove: For every fixed positive integer k, $1^k+2^k+\ldots n^k$ is $\Omega(n^{k+1})$.

3) Someone suggested that the reason you learn bubblesort is that it is good if the list is "almost sorted." To formalize this notion, we can say that the number of inversions (I(L)) in a list L is the number of pairs of elements x,y such that x comes before y in L, but x is greater than y.

Show that bubblesort does not run in O(n+I(L)) time.

4) Show that insertion sort runs in O(n+I(L)) time. Recall that insertion sort adds the next element x at the back of the list, and swaps the element with the previous element until the previous element is smaller than x.